

On the rare $B_s \rightarrow \mu\mu$ decay and noncontractibility of the physical space

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Abstract

It is very well known that the rare electroweak processes could be very sensitive to the physics beyond the Standard Model. These processes are described with quantum loop diagrams containing also heavy particles. We show that the electroweak theory with the noncontractible space, as a symmetry-breaking mechanism without the Higgs scalar, essentially changes the Standard Model prediction of the branching ratio of the B_s meson decaying to two muons. The branching ratio is lower by more than 30% compared with the Standard Model result. Although the measurements are very challenging, the implications on the selection of the symmetry-breaking mechanism could be decisive.

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I. INTRODUCTION AND MOTIVATION

In the previous paper [1], we show that the CP violation in the K and B meson systems is affected by the short distance electroweak (EW) corrections of the BY theory [2] that differ substantially from the Standard Model (SM) ones. We also show that the QCD in noncontractible space (BY is the ultraviolet nonsingular theory) enhances the standard QCD t-quark charge asymmetry and the effect is observed at the Tevatron [3]. It is natural to expect the deviation from the SM predictions for the rare electroweak processes which are described by the similar loop diagrams.

In this paper, we concentrate on the evaluation of the short distance EW corrections in the SM and the BY theory to the decay $B_s \rightarrow \mu\mu$. In the next chapter, we present the SM and BY results for the effective transition operator consisting of the Z-boson penguin and box diagram contributions [4]. The last chapter is devoted to the numerical evaluations and discussion.

II. $B_s \rightarrow \mu\mu$ AMPLITUDE

The SM calculation of the $B_s \rightarrow \mu\mu$ decay can be found in Ref.[4] and the QCD correction in Ref.[5].

The branching ratio for the $B_s \rightarrow \mu\mu$ decay is equal to [5]:

$$Br(B_s \rightarrow \mu\mu) = \tau(B_s) \frac{G_F^2}{\pi} \left(\frac{\alpha_e}{4\pi \sin^2 \Theta_W} \right)^2 F_{B_s}^2 m_\mu^2 m_{B_s} \sqrt{1 - 4 \frac{m_\mu^2}{m_{B_s}^2}} \times |V_{ts}^* V_{tb}|^2 Y^2(x_t), \quad (1)$$

$$x_t \equiv \left(\frac{m_t}{M_W} \right)^2, \quad Y(x_t) = \eta_Y Y_0(x_t), \quad \eta_Y = 1.012 = QCD \text{ correction factor},$$

$$\langle 0 | (\bar{s}b)_{V-A,\nu} | B_s(p) \rangle = i F_{B_s} p_\nu, \quad Y_0(x) = \frac{x}{8} \left(\frac{4-x}{1-x} + \frac{3x}{(1-x)^2} \ln x \right).$$

Thus, the short distance part is hidden in the gauge invariant term $Y(x_t)$ [4, 5]. We must perform the calculation within the BY theory [1, 2]. We choose the 't Hooft-Feynman gauge to make direct comparison with the SM results [4]. Although the Nambu-Goldstone scalars do not couple to Dirac fermions in the BY theory, we can use them as the auxiliary fields

in our calculus. The gauge invariant observables could be evaluated in the unitary gauge without the Nambu-Goldstone scalars.

The new ingredient of the BY theory is the UV cutoff in the spacelike domain of the Minkowski spacetime [2]: $\Lambda = \frac{\hbar}{cd} = \frac{2}{g\sqrt{6}}M_W \simeq 326 GeV$. Λ is fixed by the Wick's theorem and the trace anomaly with the Lorentz and gauge invariant weak coupling and the weak gauge boson mass.

Because of the conservation of the electromagnetic current of the muon pair, there is no γ penguin contribution to $B_s \rightarrow \mu\mu$. The Z boson penguin (Γ_Z) and the box (C) diagram contribution (see Figs.(1-4) and Appendix of Ref.[4]) have the following expressions (with the abbreviation $\sin \Theta_W = s_W$):

$$Y_0(x_j) = \frac{1}{2}(\Gamma_Z + C)(x_j) - (x_j \rightarrow x_u), \quad j = c, t, \quad \Gamma_Z \equiv \sum_{i=a}^h \Gamma^{(i)}, \quad (2)$$

$$b\bar{s}Z \text{ vertex} = \Gamma_{Z\mu}^{(i)} = \frac{1}{(4\pi)^2} \frac{g^3}{\cos \Theta_W} V_{js}^* V_{jb} \bar{s} \gamma_\mu P_L b \Gamma^{(i)}, \quad (3)$$

$$\begin{aligned} \Gamma^{(a+b)} &= \left(-\frac{1}{2} + \frac{1}{3}s_W^2\right)\left(1 + \frac{1}{2}\frac{m_j^2}{M_W^2}\right)B_1(0; m_j, M_W), \\ \Gamma^{(c)} &= -\frac{1}{4}\left(-1 + \frac{4}{3}s_W^2\right)(B_0(0; m_j, M_W) + m_j^2 L(m_j, M_W)) + \frac{2}{3}s_W^2 m_j^2 L(m_j, M_W), \\ \Gamma^{(d)} &= -\frac{1}{2}\frac{m_j^2}{M_W^2}\left(\frac{1}{3}s_W^2 B_0(0; m_j, M_W) + m_j^2\left(\frac{1}{2} - \frac{1}{3}s_W^2\right)L(m_j, M_W)\right), \\ \Gamma^{(e)} &= -\frac{3}{2}(1 - s_W^2)(B_0(0; m_j, M_W) + M_W^2 L(M_W, m_j)), \\ \Gamma^{(f+g)} &= -s_W^2 m_j^2 L(M_W, m_j), \\ \Gamma^{(h)} &= \frac{1}{8}(-1 + 2s_W^2)\frac{m_j^2}{M_W^2}(B_0(0; M_W, m_j) + M_W^2 L(M_W, m_j)), \\ C &= \frac{1}{2}M_W^2 L(M_W, m_j), \quad x_j \equiv \left(\frac{m_j}{M_W}\right)^2. \end{aligned} \quad (4)$$

The external fermion masses and momenta are neglected, while neutrinos are assumed to be massless. We can compare our expressions with those of Inami and Lim, term by term, acknowledging the following Green functions (only real parts are considered, see [2]):

$$\frac{i}{16\pi^2} B_0(k^2; m_1, m_2) \equiv (2\pi)^{-4} \int d^4 q (q^2 - m_1^2)^{-1} ((q+k)^2 - m_2^2)^{-1},$$

$$\begin{aligned}
\frac{i}{16\pi^2} k_\mu B_1(k^2; m_1, m_2) &\equiv (2\pi)^{-4} \int d^4 q q_\mu (q^2 - m_1^2)^{-1} ((q+k)^2 - m_2^2)^{-1}, \\
\frac{i}{16\pi^2} L(m_1, m_2) &\equiv (2\pi)^{-4} \int d^4 q (q^2 - m_1^2)^{-2} (q^2 - m_2^2)^{-1}, \\
B_1(0; m_1, m_2) &= \frac{1}{2} (-B_0(0, m_1, m_2) + (m_2^2 - m_1^2) \frac{\partial B_0}{\partial k^2}(0, m_1, m_2)), \\
B_0(0; m_j, M_W) &= \Delta_{UV} - x_j \ln x_j / (x_j - 1), \quad \Delta_{UV} \equiv UV \text{ infinity}, \\
\frac{\partial B_0}{\partial k^2}(0; m_1, m_2) &= \frac{1}{2} \frac{m_1^2 + m_2^2}{(m_1^2 - m_2^2)^2} - \frac{m_1^2 m_2^2}{(m_2^2 - m_1^2)^3} \ln \frac{m_2^2}{m_1^2}, \\
m_j^2 L(m_j, M_W) &= x_j (x_j - 1)^{-2} (1 - x_j + \ln x_j).
\end{aligned}$$

Adding up all graphs, we get:

$$\begin{aligned}
\Gamma_Z(x_j) &= \frac{1}{48} (x_j - 1)^{-2} \{ -[(x_j - 1)(66 + 9x_j - 15x_j^2 + 2s_W^2(-34 + 29x_j + 5x_j^2))] \\
&\quad + 12x_j(2 + 3x_j) \ln x_j \}, \tag{5}
\end{aligned}$$

$$C(x_j) = \frac{1}{2} \left(\frac{1}{x_j - 1} - \frac{x_j}{(x_j - 1)^2} \ln x_j \right). \tag{6}$$

It seems that Inami and Lim [4] make an approximation for the total Z-boson penguin contribution. Our expressions for any single graph (a-h) coincide with the Inami-Lim expressions ($A \cdot 1$) of Ref.[4]. Their total Z-boson contribution is:

$$\Gamma_Z(x_j)_{approx} = \frac{1}{4} x_j - \frac{5}{4} \frac{1}{x_j - 1} + \frac{1}{4} \frac{3x_j^2 + 2x_j}{(x_j - 1)^2} \ln x_j. \tag{7}$$

It is easy to get necessary Green functions with the UV cutoff of the BY theory from the preceding integral representations:

$$\begin{aligned}
B_0^\Lambda(0; m_1, m_2) &= \int_0^{\Lambda^2} dy \frac{y}{(y + m_1^2)(y + m_2^2)} \\
&= (m_1^2 \ln \frac{\Lambda^2 + m_1^2}{m_1^2} - m_2^2 \ln \frac{\Lambda^2 + m_2^2}{m_2^2}) / (m_1^2 - m_2^2), \\
\frac{\partial B_0^\Lambda}{\partial k^2}(0; m_1, m_2) &= \frac{1}{2} \left(\frac{\partial \tilde{B}_0^\Lambda}{\partial k^2}(0; m_1, m_2) + \frac{1}{2} \left(\frac{\partial \tilde{B}_0^\Lambda}{\partial k^2}(0; m_2, m_1) \right) \right), \\
\frac{\partial \tilde{B}_0^\Lambda}{\partial k^2}(0; m_1, m_2) &= m_2^2 \int_0^{\Lambda^2} dy \frac{y}{(y + m_1^2)(y + m_2^2)^3} \\
&= [\Lambda^2 (-2m_1^2 m_2^4 + 2m_2^6 + \Lambda^2 (m_2^4 - m_1^4)) + 2m_1^2 m_2^2 (\Lambda^2 + m_2^2)^2 \\
&\quad \times (\ln \frac{m_1^2}{m_2^2} - \ln \frac{\Lambda^2 + m_1^2}{\Lambda^2 + m_2^2})] / [2(\Lambda^2 + m_2^2)^2 (m_2^2 - m_1^2)^3],
\end{aligned}$$

$$\begin{aligned}
L^\Lambda(m_1, m_2) &= - \int_0^{\Lambda^2} dy y (y + m_1^2)^{-2} (y + m_2^2)^{-1} \\
&= - [\Lambda^2(m_1^2 - m_2^2) + m_2^2(\Lambda^2 + m_1^2) (\ln \frac{m_2^2}{m_1^2} + \ln \frac{\Lambda^2 + m_1^2}{\Lambda^2 + m_2^2})] \\
&\quad \times [(\Lambda^2 + m_1^2)(m_1^2 - m_2^2)^2]^{-1}.
\end{aligned}$$

Now we have all ingredients to make numerical estimates and comparisons between SM and BY short distance parts of the rare $B_s \rightarrow \mu\mu$ decay.

III. RESULTS AND DISCUSSION

With insertion of the Green functions, with and without the UV cutoff, into the expressions for the amplitude, one can compare branching ratios. We can safely ignore c-quark contribution in the amplitude since $|V_{cb}^* V_{cs} Y(x_c)| / |V_{tb}^* V_{ts} Y(x_t)| \simeq \mathcal{O}(10^{-4})$. Let us numerically inspect the difference between our exact and Inami-Lim approximate sum for the Z-boson penguin (Eqs.(5) and (7)):

$$\begin{aligned}
\text{parameters : } s_W^2 &= 0.23, \quad m_u = 3MeV, \quad m_c = 1.3GeV, \quad m_t = 172GeV, \\
M_W &= 80.4GeV, \quad \Lambda = 326GeV \rightarrow \Gamma_Z(x_t)_{approx} / \Gamma_Z(x_t) = 0.962.
\end{aligned}$$

We see that the difference is only a few percents.

Since the QCD correction to the Y_0 is evaluated at the scale $\mu \simeq m_b$ much smaller than Λ , it does not deviate from the SM in the BY theory. The large uncertainty in the quark mixing angles and the meson form factor f_{B_s} is the consequence of nonperturbative QCD hadron physics. Now we make our final estimate and comparisons for the branching ratio of the $B_s \rightarrow \mu\mu$ decay:

$$\begin{aligned}
\frac{Br(B_s \rightarrow \mu\mu)(\Lambda)}{Br(B_s \rightarrow \mu\mu)(\infty)} &= \frac{Y_0^2(x_t)|_\Lambda}{Y_0^2(x_t)|_\infty} = 0.664, \\
\frac{Br(B_s \rightarrow \mu\mu)(\Lambda)}{Br(B_s \rightarrow \mu\mu)(\infty)_{approx}} &= 0.708.
\end{aligned}$$

The estimate with the UV cutoff (noncontractible space as a symmetry breaking mechanism) gives a more than 30% lower branching ratio compared to the SM. For a UV cutoff that is larger by one order of magnitude, the difference diminishes $Br(10\Lambda = 3260GeV) / Br(\Lambda = \infty) = 0.996$.

The cumulative error of the SM prediction $Br(SM) = (3.2 \pm 0.2) \cdot 10^{-9}$ is below 10% [6], thus giving the opportunity to the Tevatron, LHCb, SuperKEKB and SuperB to discriminate between the two predictions in the very near future [7].

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